Ch. 5 Dynamics---all the physics stuff

In Newtonian physics---the generalized statement regarding force --was not in terms of "ma" but instead in terms of rate of change of momentum $\vec{F} = \partial \vec{P} / \partial t$

That formulation allowed for the fact that in some processes, the mass of objects may change in addition to the possibility of their velocity changing. In special relativity we will find that mass depends on motion---and therefore we must start with defining MOMENTUM.

$$\vec{P} = m\vec{V}$$

Momentum will use the same old definition, and it appears that momentum is conserved too.

Mass however changes when objects move!!!!!!

Force is still related to "rate of change of momentum"---
But ma needs modification.

If $F_{net}=0$ (on an object), then it is still true that momentum cannot change.

We are going to allow a collision to occur---
View it from one frame, then another.

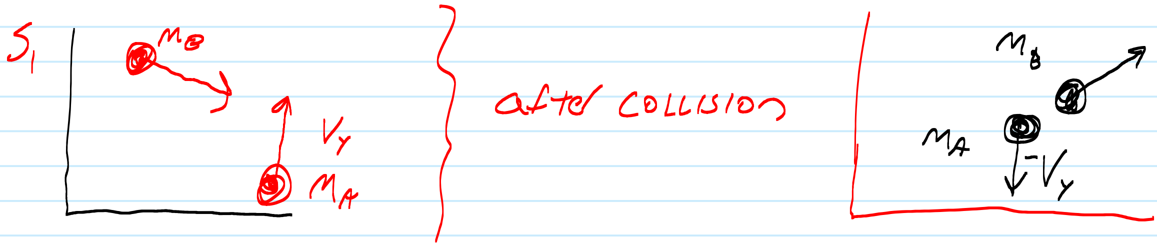
Our two reference frames 1 and 2, still have the same general relation. We will consider a special collision (still perfectly general) to simplify the components we must keep track of.

See figure 5.1 a, and b. I'll sketch it here.

In frame 1

S_2 moves $\longrightarrow V$

ball A fired up in S_1

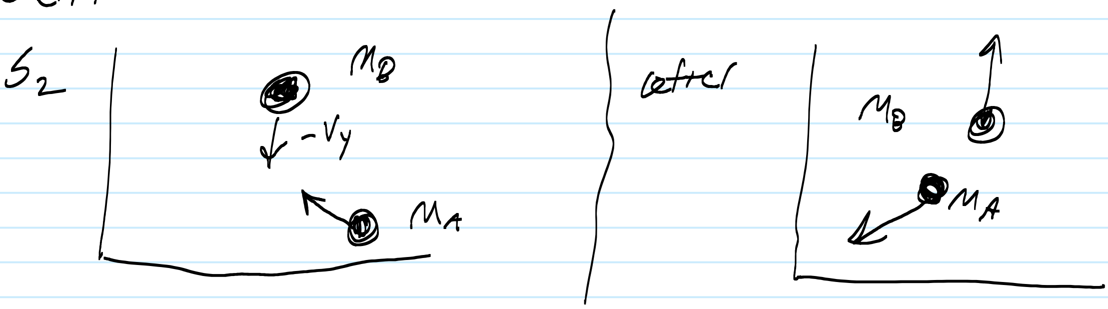


The collision is completely general-between balls A and B.
I have simply decided that frame 1 rides along with the speed so that ball A has no x component of velocity.

Frame 2 will ride along at a different speed "v" so that ball B has no x component of velocity.

I don't need z since two body collisions occur in a plane.

In Frame S_2 the same collision looks different



I have drawn before and after shots for the collision as viewed in frame 1 and also in frame 2.

The numbers for frame 1 (everything that goes into momentum conservation) are:

m_{A1}, m_{B1}, $v_{A,1x}$, $v_{A,1y}$		$v'_{A,1x}$, $v'_{A,1y}$		$v_{B,1x}$, $v_{B,1y}$		$v'_{B,1x}$, $v'_{B,1y}$

So there are 10 numbers describing the collision in frame 1. Likewise 10 (possibly different) numbers describing the collision in frame 2. We will use these to describe momentum conservation.

We get to decide to make whatever collision we want. We have(the book) picked --in frame 1---by riding with x component of velocity of ball A--there is no change in x momentum of ball A. In the y direction we are riding along so that ball A bounces back elastically at the same speed. Any "y" momentum picked up came from ball B.

Also, the firing speeds in each frame are slow enough so that the ball headed vertical can be considered "approximately at rest". ----and in their own rest frames, the balls are the same mass.

We have made a symmetric collision in each frame--and are allowed to do so.

Tables---each ball, each frame, before/after.

Ball A-before-in S_1				Ball A before in S_2

m_o $\quad V_{A1x} = 0$

$V_{A1y} = +30$ m/s

m $\quad V_{A2x} = -V$

do trans with $V_{A1x} = 0$ $\rightarrow$ $V_{A2y} = V_{A1y}\sqrt{1-\beta^2}$

Ball A -after- in S_1				Ball A after in S_2

m_o $\quad V'_{A1x} = 0$

$V'_{A1y} = -30$ m/s

m $\quad V'_{A2x} = -V$

$V'_{A2y} = V'_{A2y}\sqrt{1-\beta^2}$

trans $\quad = (-V_{A1y})\sqrt{1-\beta^2}$

Ball B S_1 before

Ball B S_2 before

m $\quad V_{B1x} = V$

$V_{B1y} = V_{B2y}\sqrt{1-\beta^2}$ $\leftarrow$

m_o $\quad V_{B2x} = 0$

$V_{B2y} = -30$ m/s

After

After

m $\quad V'_{Bx} = V$

$V'_{B1y} = V'_{B2y}\sqrt{1-\beta^2}$ $\leftarrow$

m_o $\quad V'_{B2x} = 0$

$V'_{B2y} = +30$ $\leftarrow$ is V_{A1y}

You have the information now to set up any of the 4 conservation of momentum equations x,y, in frame 1 or 2.
p(total before) =p(total after)----do this for the y equations.

In order for momentum to be conserved you must get

$$m = \gamma m_o$$

Moving masses ---get bigger.
This introduces problems as v approaches "c".

It would require infinite energy to reach c for any mass
Infinite force to have any acceleration at all---or to change momentum (of that infinite mass).

Force

$$\vec{F} = \frac{d\vec{p}}{dt}$$

But recall, p=mv---and both mass and v now depend on v. And v may depend on time for a dynamic particle.

$$= \frac{d(m\vec{v})}{dt}$$

$$= \underbrace{\frac{dm}{dt}}_{\text{we used to consider this zero}}\vec{v} + m\frac{d\vec{v}}{dt}$$

$$= \vec{v}\,\frac{dm}{dt} + m\vec{a}$$

We notice immediately that "F=ma" is out---
In special relativity the dm/dt term cannot fall away since mass depends on speed (can approximately drop if v<<c).

We will consider two cases

1) components of force that cause change in direction only
   a. When Force is applied perpendicular to motion---there is no change in speed, so dm/dt =0.
   b. The messy term is gone
2) components of force that cause change in speed.
   a. We have a change in speed and need to keep the dm/dt term.
   b. Since $m=\gamma m_o$ we can take the derivative

1) For $F_\perp$ we simply get $F_\perp = ma_\perp$

For force parallel--we need to take dm/dt

$$\frac{dm}{dt} = \frac{d}{dt}\left(\frac{m_0}{[1-v^2/c^2]^{\frac{1}{2}}}\right)$$

$$= m_0\left(-\frac{1}{2}\right)\left(1-v^2/c^2\right)^{-3/2}\left(-2v/c^2\right)\frac{dv}{dt}$$

$$= m_0\gamma^3\frac{v}{c^2}\frac{dv}{dt}$$

this is $a_{\parallel}$ in this case

$$= m_0\gamma^3\frac{v}{c^2}\,a_{\parallel}$$

You should be able to quickly check dimensions and make sure that works.

We still need to put things together for the parallel case

go back to $\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$

$$F_{\parallel} = m\frac{dv}{dt} + v\left(m_0\gamma^3\frac{v}{c^2}\,a_{\parallel}\right)$$

$$= m_0\gamma a_{\parallel} + m_0\gamma^3\frac{v^2}{c^2}\,a_{\parallel}$$

$$= m_0\gamma a_{\parallel}\left(1 + \frac{v^2}{c^2}\gamma^2\right)$$

$$= m_0\gamma a_{\parallel}\left(1 + \frac{v^2}{c^2}\frac{1}{1-v^2/c^2}\right)$$

common denom & simplify $\rightarrow \gamma^2$

$$= m_0\gamma^3 a_{\parallel}$$

We now have a full description of what happens when we apply parallel or perp components of F to an object. The response is NOT directly proportional to the acceleration--but rather.

$$F_{\parallel} = \gamma^3 m_0 a_{\parallel}$$

$$F_{\perp} = \gamma m_0 a_{\perp}$$

Force is applied, a is response

consider equal momentary $F_{\parallel}$ & $F_{\perp}$

$$a_{\parallel} = \frac{F_{\parallel}}{\gamma^3 m_0}$$

$$a_{\perp} = \frac{F_{\perp}}{\gamma m_0}$$

smaller for same force

Apparently it is easier to turn an object than to speed it up!!!!!!!!!!!!!

Take-away message. The acceleration is NOT in a direction parallel to the force.

These are not transformations--but are simply fixes on the previous thing that we wrongly called Newton's 2nd law. The real law is properly stated with momentum conservation. The inclusion of the mass term means we have rewritten what one observer measures for "a" as a force is applied (remote controlled constant thrust rocket ---aimable). .....

Energy

Again we revisit our statements and definitions ---and we do not transform, but merely ensure that our old definitions are fixed up.

If we start to push an object from rest--we do work on the object and it picks up Kinetic Energy .

general $(d\vec{l})$

$$KE = \int_{Path} \vec{F} \cdot d\vec{x}$$

$F_\perp$ does no work

on mass m_0

$$= \int_{Path} F_{\parallel}\, dx$$

Consider as one dim

$$= \int \gamma^3 m_0 a \, dx \qquad //$$

$$= m_0 \int \left(1 - v^2/c^2\right)^{-3/2} \frac{dv}{dt}\, dx$$

Note if v<<c then the v^2/c^2 drops out, and our integral is simply v dv ---so we get normal classical $(1/2)mv^2$ for slow speeds.

If v is significant compared to speed of light---

$$KE = m_0 \int_0^v \left(1 - v'^2/c^2\right)^{-3/2} v' \, dv'$$

we made the $\int$ definite (1+ 15)

OK do the integral ... Lookup....

$$= m_0 c^2 \left(1 - v^2/c^2\right)^{-1/2} - m_0 c^2$$

$$= (\gamma - 1) m_0 c^2$$

$$= \Delta m \, c^2$$

Where

$$\Delta m = m - m_0 = \gamma m_0 - m_0$$

If we want to change the kinetic energy of an already moving object, then we simply have m_i and m_f

We may consider the object at rest to have rest energy

$$E_0 = m_0 c^2$$

In motion $E = m c^2$

$$KE = E - E_0$$

Again check to reduce to non-rel.
Again recall
$$(\gamma - 1) m_0 c^2$$

$E=mc^2$

$= \gamma m_o c^2$

$E_0 = m_o c^2$ and $KE = E - E_o$

$= (\gamma-1) m_o c^2$

Any process changing the energy of a particle, or system---changes it's mass.

Like Example 5.8

If a proton is given 500 MeV of kinetic energy — what is E total

$$E_0 = m_0 c^2$$
$$= 1.67 \times 10^{-27}\ kg \times (3.00 \times 10^8\ m/s)^2$$
$$= 1.50 \times 10^{-10}\ J$$
$$= 938\ MeV$$

$$E_{TOT} = 1438\ MeV$$

$$m_{Tot} = E_{TOT} / c^2$$

In the last example, we can also ask---

- what is γ ,
- and what is the speed,
- how far off is the classical KE (which is incorrect) from the relativistic KE ?

We have already considered conservation of momentum with forces---Newton's 2nd law, and it is unchanged except for the inclusion of dp/dt and change in mass terms.

However---now we have a new way to write kinetic energy.

An "old" way to write KE was $\frac{1}{2} m v^2$

or $p^2/2m$

The new way is $(\gamma - 1)\, m_0 c^2$ γ has v

this reduces for $v \ll c$

Consider how momentum (magnitude) and Energy (mass) relate.

$$m = \gamma m_0$$

$$= m_0 \big/ (1 - v^2/c^2)^{\frac{1}{2}}$$

squared & Rearrange

$$m^2 (c^2 - v^2) = m_0^2 c^2$$

$$E^2 = p^2 c^2 + E_0^2$$

$|p| = |mv|$

$E_0 = m_0 c^2$

$E = mc^2$

This form--allows us to consider what happens with particles having ZERO REST MASS.

$$E^2 - E_o^2 = p^2c^2$$

$$\underbrace{(E - E_o)}_{KE}\underbrace{(E + E_o)}_{KE + E_o} = p^2c^2$$

We can consider slow moving particles (KE<<E or E_o)
Fast moving particles (does this mean KE~E .....or KE>> E_o)
Massless particles E_o=0

Wait a minute---do massless particles carry Energy, momentum?

Apparently yes to both.